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First order gauge field theories from a superfield formulation

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Abstract: Recently, Batalin and Marnelius proposed a superfield algorithm for master actions in the BV-formulation for a class of first order gauge field theories. Possible theories are determined by a ghost number prescription and a simple local master equation. We investigate consistent solutions of these local master equations with emphasis on four and six-dimensional theories.

Keywords: BRST Quantization, Topological Field Theories, Gauge Symmetry.

Contents

1. Introduction

The Batalin-Vilkovisky (BV) formalism [1, 2] provides a powerful framework for construction of quantum gauge field theories. In this procedure the fundamental object is the so called master action, consisting of fields and their antifields. A necessary consistency condition is that this master action satisfies the master equation. Recently it has been shown that many models allow for a simple form of the master action in terms of superfields [3]–[17]. In this paper we will follow the approach considered in [16, 17]. There Batalin and Marnelius introduced a superfield algorithm for a class of first order gauge field theories from a master action written in terms of superfields. This type of master action was originally considered in [9, 12]. By means of a ghost number prescription this algorithm provides for a simple and powerful method of finding all possible (local) terms entering in such a master action. Furthermore, with appropriate boundary conditions this master action can be expressed in terms of a local master action satisfying a much simpler local master equation.

This superfield algorithm leads to many consistent theories, including the two-dimensional Poisson sigma model [18, 19] and its superfield formulation considered by Cattaneo and Felder [8, 11], Chern-Simons, topological Yang-Mills, BF-theories and generalizations thereof. It also agrees with the general framework set up by AKSZ [12].

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In this communication we construct new consistent solutions of the local master equation using (anti)canonical transformations. We give explicit solutions to the master equation for first order gauge field theories in four and six dimensions. In several papers [10] and [20]–[26] it has been established that a large class of theories are just trivial deformations of theories of a simpler kind. Our results in six dimensions support these results. It seems that only two interaction terms can account for a large class of first order theories in six dimensions. Many of the other possible terms (according to the superfield algorithm) belongs to the same equivalence class and can thus be obtained by (anti)canonical transformations from these two terms. We also discuss some general feature of this class of gauge field theories in various dimensions.

The paper is organized as follows. In the next section we give a short review of the procedure proposed by Batalin and Marnelius [16, 17]. Section 3 discusses the construction of solutions to the master equation, for the class of first order gauge field theories considered in section 2, using (anti)canonical transformations. In section 4 this construction is applied to six-dimensional theories. In section 5 we discuss how one can derive the gauge transformations of the classical model that can be extracted from a specific master action. A discussion of some general features in various dimensions of the class of quantum field theories studied in this paper is then given in section 6. In the appendices we give notations, conventions and some special results.

2. Preliminaries

In [16, 17] Batalin and Marnelius proposed a superfield algorithm for the construction of master actions for a class of first order gauge field theories. The master action Σ corresponding to such an n-dimensional theory can be written as a field theory living on a $2n$-dimensional supermanifold $\mathcal{M}$, where n of the dimensions are Grassmann odd and n Grassmann even,

$$\Sigma\left[K^P, K^*_P\right] = \int_{\mathcal{M}} d^n u d^n \tau \mathcal{L}_n(u,\tau)\,, \tag{2.1}$$

where K^P and K^*_P are superfields. The supermanifold $\mathcal{M}$ is coordinatized by (u^a, τ^a), where $a = \{1, \ldots, n\}$ and u^a denotes the Grassmann even and τ^a the Grassmann odd coordinates, respectively. The lagrangian density $\mathcal{L}_n$ is of the form

$$\mathcal{L}_n(u,\tau) = K^*_P(u,\tau) D K^P(u,\tau)(-1)^{\epsilon_P + n} - S\left(K^*_P(u,\tau), K^P(u,\tau)\right), \tag{2.2}$$

where D is the de Rham differential which locally can be written as

$$D := \tau^a \frac{\partial}{\partial u^a}\,. \tag{2.3}$$

It follows that D is nilpotent

$$D^2 = 0\,. \tag{2.4}$$

Note that the terms in $S(K^*_P(u,\tau), K^P(u,\tau))$ only contains superfields without derivatives. The parities of the superfields K^P and associated superfields K^*_P are given by

$$\begin{aligned} \epsilon\left(K^P\right) &:= \epsilon_P\,, \\ \epsilon(K^*_P) &= \epsilon_P + 1 + n\,. \end{aligned} \tag{2.5}$$

Since the de Rham differential D and τ^a carries ghost number one, the measure $d^n\tau$ has ghost number-n. This implies, since the master action Σ has ghost number zero, that

$$gh_{\#}K^P + gh_{\#}K^*_P = n-1\,. \tag{2.6}$$

The local function S possesses the following ghost number and Grassmann grading

$$\begin{aligned} gh_{\#}(S) &= n\,, \\ \epsilon(S) &= n\,. \end{aligned} \tag{2.7}$$

The equations of motion that follows from the master action Σ in (2.1) are

$$DK^P = \left(S, K^P\right)_n, \qquad DK^*_P = (S, K^*_P)_n\,. \tag{2.8}$$

They provide for a natural BRST-charge operator interpretation of the de Rham differential, defined in (2.3). The local n-bracket $(\ ,\ )_n$ introduced above, is defined by

$$(F,G)_n = F\frac{\overleftarrow{\partial}}{\partial K^P}\frac{\overrightarrow{\partial}}{\partial K^*_P}G - (F \longleftrightarrow G)(-1)^{(\epsilon(F)+n+1)(\epsilon(G)+n+1)}\,. \tag{2.9}$$

It satisfies the Jacobi identity

$$((F,G)_n, H)_n(-1)^{(\epsilon(F)+n+1)(\epsilon(G)+n+1)} + \text{cycle}\,(F,G,H) = 0\,, \tag{2.10}$$

and has the graded symmetry property

$$(F,G)_n = -(-1)^{(\epsilon(F)+n+1)(\epsilon(G)+n+1)}(G,F)_n\,. \tag{2.11}$$

Due to (2.10) and (2.11) we observe that $(\ ,\ )_n$ is an "ordinary" antibracket in even dimensions and a super Poisson bracket in odd dimensions. The n-bracket carries 1-n units of ghost number

$$gh_{\#}(F,G)_n = gh_{\#}F + gh_{\#}G + 1 - n\,, \tag{2.12}$$

and n+1 units of parity

$$\epsilon((F,G)_n) = \epsilon(F) + \epsilon(G) + n + 1\,. \tag{2.13}$$

It also satisfies the Leibniz rule

$$\begin{aligned} (F,GH)_n &= (F,G)_nH + G(F,H)_n(-1)^{\epsilon(G)(\epsilon(F)+n+1)}\,, \\ (FG,H)_n &= F(G,H)_n + (F,H)_nG(-1)^{\epsilon(G)(\epsilon(H)+n+1)}\,. \end{aligned} \tag{2.14}$$

Notice that the respective expansion of the superfields K^P and K^*_P in terms of the odd coordinates τ^a lead to component fields which are either fields or antifields. Since the original fields constitute the ghost number zero components of the superfields K^P and

K^*_P, one obtains the following rules for extracting the n-dimensional classical field theory corresponding to a given master action Σ of the form (2.1)

$$\begin{aligned} d^n u d^n \tau &\longrightarrow 1\,, \\ D &\longrightarrow \text{exterior derivative } d\,, \\ K^P : gh_{\#} K^P = k \geq 0 &\longrightarrow k\text{-form field } k^P \text{ where, } \epsilon\left(k^P\right) = \epsilon_P + k\,, \\ K^*_P : gh_{\#} K^*_P = (n-1-k) \geq 0 &\longrightarrow (n-1-k) - \text{form field } k^*_P \text{ where, } \epsilon(k^*_P) = \epsilon_P + k\,, \\ \text{all other superfields} &\longrightarrow 0\,, \\ \text{pointwise multiplication} &\longrightarrow \text{wedge product}\,. \end{aligned} \tag{2.15}$$

3. Solutions to the master equation

In this section we consider the construction of theories having a master action of the form [16, 17],

$$\Sigma\left[K^*_P, K^P\right] = \int d^n u d^n \tau K^*_P(u,\tau) D K^P(u,\tau)(-1)^{\epsilon_P + n} - S\left(K^*_P(u,\tau), K^P(u,\tau)\right).$$

The quantum master equation for a model Σ is given by,

$$\frac{1}{2}(\Sigma,\Sigma) = i\hbar\Delta\Sigma\,, \tag{3.1}$$

where the antibracket (A,B) between two functionals is defined by,

$$\begin{aligned} (A,B) := \int A \frac{\overleftarrow{\delta}}{\delta K^P(u,\tau)}(-1)^{(\epsilon_P n)} d^n u\, d^n \tau \frac{\overrightarrow{\delta}}{\delta K^*_P(u,\tau)} B - \\ - (A \longleftrightarrow B)(-1)^{(\epsilon(A)+1)(\epsilon(B)+1)}\,, \end{aligned} \tag{3.2}$$

and where the BV-laplacian is given by,

$$\Delta = \int d^n u d^n \tau (-1)^{(n+1)\epsilon_P} \frac{\overrightarrow{\delta}}{\delta K^P(u,\tau)} \frac{\overrightarrow{\delta}}{\delta K^*_P(u,\tau)}\,. \tag{3.3}$$

In [17] it was shown that Σ fulfills the quantum master equation, provided that S satisfies the local *n-bracket master equation*,

$$(S,S)_n = 0\,, \tag{3.4}$$

and the boundary condition

$$\int d^n u d^n \tau D\mathcal{L}(u,\tau) = 0\,. \tag{3.5}$$

Only when (3.4) and (3.5) are satisfied does Σ represent a master action for a theory that may be quantized in a consistent way. One may note that for example Dirichlet boundary values on all the superfields trivially satisfies (3.5), but more general situations are also allowed for.

Given a specific ansatz for the local master action S, the problem is to find explicit expressions for the coefficients of the superfields in S that solves (3.4). Finding such solutions by inserting a generic ansatz for S into the master equation are in general very difficult since it might lead to equations for the coefficients in the ansatz that are too complicated to solve. The general idea here is instead to start with a local master action S_0 for which the solution is known, and then perform canonical transformations[1] of S_0. If the local action S_0 is chosen to have exactly the same superfield content as the original model S, the canonically transformed S_0 will in several cases constitute a solution for S. We will see examples of this below.

A general (invertible) canonical transformation of an object F is given by

$$F_\Gamma = e^{\mathrm{ad}\,\Gamma} F \,, \tag{3.6}$$

where Γ is a canonical generator with adjoint action

$$\mathrm{ad}\,\Gamma = (\cdot, \Gamma)_n \,. \tag{3.7}$$

Instead of solving the local master eq. (3.4) directly we propose now to use the canonical transformation

$$S_\Gamma = e^{\gamma\,\mathrm{ad}\,\Gamma} S_0 = S_0 + \gamma (S_0, \Gamma)_n + \frac{\gamma^2}{2!} \left((S_0, \Gamma)_n \,, \Gamma \right)_n + \cdots \,, \tag{3.8}$$

where γ denotes a real and even parameter. In terms of papers [23, 24] γ is the deformation parameter (in eq. (3.8) we have chosen to display it explicitly instead of absorbing it into Γ). Canonical transformations correspond to trivial deformations — which implies that they do not change the gauge structure. We have then

$$(S_0, S_0)_n = 0 \qquad \Longrightarrow \qquad (S_\Gamma, S_\Gamma)_n = 0 \,. \tag{3.9}$$

Since Γ is a canonical generator, it must preserve the ghost number — and Grassmann gradings of the transformed quantity. From the properties (2.12) and (2.13) of the n-bracket, it follows then that we must impose the restrictions

$$\begin{aligned} gh_{\#}\Gamma &= n - 1 \,, \\ \epsilon(\Gamma) &= n + 1 \quad \pmod 2 \,. \end{aligned} \tag{3.10}$$

To be able to write down closed expressions for the canonically transformed quantities we consider a subset of canonical transformations, for which (3.8) always consists of a finite sum. To achieve this we consider the following form of the generators that always will generate a canonical transformation of the superfields which is first order in the parameter γ (see appendix C)

$$\Gamma = K^*_{P_1} K^*_{P_2} \ldots K^*_{P_n} \Gamma^{P_1 P_2 \ldots P_n}{}_{P_{1'} P_{2'} \ldots P_{n'}} K^{P_{1'}} K^{P_{2'}} \ldots K^{P_{n'}} \,, \tag{3.11}$$

where

$$\forall P_i, P_{i'} : P_i \neq P_{i'} \,. \tag{3.12}$$

[1]When mentioning canonical transformations in this paper, it is to be understood that we mean the transformations that preserves the local n-bracket, which are either canonical or anticanonical depending on n.

K^P and K_P^* are collective labels for all superfields. The coefficients in (3.11), $\Gamma^{P_1 P_2 \dots P_n}{}_{P_{1'}}{}_{P_{2'} \dots P_{n'}}$, may be functions of the ghost number zero superfields. Above Γ is chosen not to contain both a specific superfield and its associated superfield simultaneously. Note that the superfields in Γ must be chosen to satisfy the conditions in (3.10).

Note that since the master action Σ_0 in (2.1) transforms canonically as

$$\begin{aligned} \Sigma_{\bar{\Gamma}} &= e^{\mathrm{ad}\,\bar{\Gamma}} \Sigma_0 = \int d^n u d^n \tau \, e^{\mathrm{ad}\,\Gamma} \mathcal{L}_n \\ &= \int d^n u d^n \tau \, K_P^* D K^P (-1)^{\epsilon_P + n} - e^{\mathrm{ad}\,\Gamma} S_0 + D\Gamma \,, \end{aligned} \tag{3.13}$$

it suffices to study canonical transformations of the local action S_0, provided $\int d^n u d^n \tau \, D\Gamma = 0$. Above, $\bar{\Gamma} = \int d^n u d^n \tau \, \Gamma$ and Γ is the local canonical generator defined by the properties in (3.10), (3.11) and (3.12). Since $\bar{\Gamma}$ is a functional, $\mathrm{ad}\,\bar{\Gamma}$ is defined in terms of the functional antibracket (3.2).

S_0 can often be written with only a few terms in such a way that it trivially satisfies the local master equation. A suitable choice of S_0 will be transformed to a S_Γ containing terms allowed by the general ansatz for S, and as is shown in the examples given below, this can be done for general forms of S. By comparing the coefficients in S and S_Γ, the explicit solution to the local master equation can be read off directly. In this way solutions to very involved models in various dimensions can be generated.

Before moving on to six-dimensional gauge field theories in the next section, let us illustrate the method for a class of models in four dimensions.

3.1 Topological field theory in $n = 4$

In a previous work by Cattaneo and Felder [8] the quantization of the Poisson sigma model was studied. Their results were later generalized by Batalin and Marnelius [16] in the superfield formulation of BV quantization considered here. When they then generalized the method to any dimension in [17] they considered as an application models in $n = 4$ that constitute a class of theories similar to Topological Yang-Mills theories. Here we show that this theory is canonically equivalent to a 2-form self interacting type of theory in $n = 4$. The model considered in [17] is defined by the local action

$$S = \frac{1}{2} T_{E_1}^* T_{E_2}^* \omega^{E_1 E_2} + \frac{1}{2} T_{E_1}^* \omega^{E_1}{}_{E_2 E_3} T^{E_2} T^{E_3} + \frac{1}{24} \omega_{E_1 E_2 E_3 E_4} T^{E_1} T^{E_2} T^{E_3} T^{E_4} \,, \tag{3.14}$$

where the superfields carries the following ghost numbers and parities:

$$\begin{aligned} gh_{\#}\left(T^E\right) &= 1 \,, \\ gh_{\#}(T_E^*) &= 2 \,, \\ \epsilon\left(T^E\right) &= 1 \,, \\ \epsilon(T_E^*) &= 0 \,. \end{aligned} \tag{3.15}$$

In order to find a solution to the local master equation for S, i.e. $(S, S)_n = 0$, we consider the term,

$$S_0 = T_{E_1}^* T_{E_2}^* \omega^{E_1 E_2} \,. \tag{3.16}$$

Obviously S_0 satisfies $(S_0, S_0)_4 = 0$. The only possible[2] canonical generator for $n = 4$ which contains only positive ghost number superfields is,

$$\Gamma = \frac{1}{3}\gamma\,\gamma_{E_1E_2E_3}T^{E_1}T^{E_2}T^{E_3}\,. \tag{3.17}$$

Above γ is real and parametrizes the canonical transformation. Due to the parity of the fields, and since

$$\begin{aligned}\epsilon(S_0) &= 0\,,\\ \epsilon(\Gamma) &= 1\,,\end{aligned} \tag{3.18}$$

we have the following Grassmann gradings and symmetry properties for the coefficients:

$$\begin{aligned}\epsilon(\gamma_{E_1E_2E_3}) &= 0\,,\\ \epsilon\left(\omega^{E_1E_2}\right) &= 0\,,\\ \omega^{E_1E_2} &= \omega^{E_2E_1}\,,\\ \gamma_{E_1E_2E_3} &= -\gamma_{E_1E_3E_2} = \gamma_{E_3E_1E_2}\,.\end{aligned} \tag{3.19}$$

The canonically transformed action is given by

$$\begin{aligned}S_\Gamma =\ & T^*_{E_1}T^*_{E_2}\omega^{E_1E_2} - 2\gamma T^*_{E_1}\omega^{E_1E}\gamma_{EE_2E_3}T^{E_2}T^{E_3} +\\ & +\gamma^2\gamma_{E_1E_2E'}\omega^{E'E}\gamma_{EE_3E_4}T^{E_1}T^{E_2}T^{E_3}T^{E_4}\,.\end{aligned} \tag{3.20}$$

A comparison with (3.14) yields the following solution to the master equation,

$$\omega^{E_1}{}_{E_2E_3} = -4\gamma\omega^{E_1E}\gamma_{EE_2E_3}\,, \tag{3.21}$$

$$\omega_{E_1E_2E_3E_4} = 24\gamma^2\gamma_{E_1E_2E}\omega^{EE'}\gamma_{E'E_3E_4}\,. \tag{3.22}$$

These solutions satisfy the master equation by construction — as one may easily check by inserting them into to the equations generated by $(S,S)_4 = 0$. The master equation for the action (3.14) actually gives three equations — one of which is enforcing the Jacobi-identities on the coefficients $\omega^{E_1}{}_{E_2E_3}$, which implies that they in general belong to a super Lie algebra. From (3.21) we see that if $\omega^{E_1E_2}$ is invertible, we may interpret it as a group metric. Lowering the indices in eq. (3.21) implies that $\omega_{E_1E_2E_3}$ will be proportional to $\gamma_{E_1E_2E_3}$ and thus totally antisymmetric with respect to all its indices — this agrees with the fact that $\omega_{E_1E_2E_3}$ then belong to a semi-simple Lie algebra. This is manifest in the solution above.

Equations (3.21) and (3.22) display a generic feature of solutions obtained in this way — all the generated coefficients will be polynomials in terms of the coefficients used in the canonical transformation and the coefficients in S_0. One should also observe that this is also a solution to the more general case when all the coefficients in eq. (3.14) as well as $\gamma_{E_1E_2E_3}$ are general functions of some ghost number zero superfields. This will

[2]For sakes of simplicity we only consider canonical transformations that are non vanishing in the limit (2.15).

neither affect the master equation nor the canonical transformations. The solution above obviously degenerates when $\omega^{E_1 E_2} = 0$ but in this case the coefficient will not be present in the original model (3.14) either. In such a situation one could use another S_0, for example $S_0 = \frac{1}{2} T^*_{E_1} \omega^{E_1}{}_{E_2 E_3} T^{E_2} T^{E_3}$, and proceed along the same lines to find a solution. In the case of a non-invertible $\omega^{E_1 E_2}$, a group metric interpretation is not possible. Hence, S_0 in eq. (3.16) is only canonically equivalent to models whose coefficients can be factorized according to (3.21) and (3.22). A careful analysis of each given model (3.14) is required in order to decide whether this is the case or not.

We observe that since the generator (3.17) is the only one that contains exclusively superfields with positive ghost number and that does not generate the identity transformation, it follows trivially that the Lie-algebra term

$$T^*_{E_1} \omega^{E_1}{}_{E_2 E_3} T^{E_2} T^{E_3} \tag{3.23}$$

is canonically inequivalent to the 2-form self interacting type of term,

$$T^*_{E_1} T^*_{E_2} \omega^{E_1 E_2} \,. \tag{3.24}$$

Canonical inequivalence is however a necessary but *not* sufficient condition for establishing the fact that two theories have different gauge structure — for example, pure Maxwell theory and Born-Infeld theory are canonically inequivalent, but do possess the same gauge structure [27].

4. $n = 6$ gauge field theories

Consider the classical action for a general theory of Schwarz type [28, 29] in $n = 6$, i.e. a non-trivial first order classical theory whose fields can be written entirely in terms of differential forms. Such a general action consists of a linear combination of terms with form degree 6, where the form degree of the individual fields is spanning from 0 to 6. The 0-degree fields enters in the coefficients of the other fields since the latter are functions. In the construction below we will restrict ourselves to models which does not contain any fields of form degree 6 and omit theories whose master action have interaction terms[3] containing fields with ghost number 5. This is a mild restriction and the reasons for this omission is discussed at the end of this section.

Consider a classical action

$$S_{\mathrm{Cl}}[\Phi] = \int d^6 x \mathcal{L}(\Phi(x)) \,, \tag{4.1}$$

which is a general functional of fields of form degree between 0 and 6. Now we will quantize this model in the superfield formalism according to the algorithm described in [17]. The solution that we will construct is a minimal one in the sense that the corresponding master

[3]Interaction terms are those contained in S, given the master action: $\Sigma[K, K^*] = \int d^n u d^n \tau (K^* D K - S[K, K^*])$.

action Σ, which has the classical action $S_{\rm Cl}$ as a limit, contains the smallest possible number of fields. This we do for the sake of clarity. Extensions are easy to construct and these will be discussed below.

Start by introducing $\{S^*_D, S^D, R^*_C, R^C, Q^*_B, Q^B\}$ as the set of superfields, where the ghost number- and Grassmann ($\mathbb{Z}_2$) gradings of the fields are given by

	S^D	S^*_D	R^C	R^*_C	Q^B	Q^*_B
$gh_\#$	2	3	1	4	0	5
Parity	ϵ_D	ϵ_D+1	ϵ_C	ϵ_C+1	ϵ_B	ϵ_B+1

Observe that the fact $gh_\# Q^*_B = 5$ does not exclude Q^*_B from existing as a free field in the models we are studying. The most general local action that can be written down under these conditions is,

$$\begin{aligned} S = {}& \omega_{C_1C_2C_3C_4C_5C_6} R^{C_1}R^{C_2}R^{C_3}R^{C_4}R^{C_5}R^{C_6} + \\ &+ \omega_{C_1C_2C_3C_4D_1} R^{C_1}R^{C_2}R^{C_3}R^{C_4}S^{D_1} + \\ &+ \omega_{D_1D_2C_1C_2} S^{D_1}S^{D_2}R^{C_1}R^{C_2} + \omega_{D_1D_2D_3} S^{D_1}S^{D_2}S^{D_3} + \\ &+ S^*_{D_1}S^*_{D_2}\omega^{D_1D_2} + R^*_{C_1}\omega^{C_1}{}_{D_1}S^{D_1} + R^*_{C_1}\omega^{C_1}{}_{C_2C_3}R^{C_2}R^{C_3} + \\ &+ S^*_{D_1}\omega^{D_1}{}_{C_1C_2C_3}R^{C_1}R^{C_2}R^{C_3} + S^*_{D_1}\omega^{D_1}{}_{C_1D_2}R^{C_1}S^{D_2} . \end{aligned} \tag{4.2}$$

All the various coefficients ω above are functions of the ghost number zero superfield Q^B. The parities and symmetry properties of the coefficients will not be given here since they follow directly from the parity of the superfields and the fact that $\epsilon(S) = 0$. Starting with the action

$$S_0 = R^*_{C_1}\omega^{C_1}{}_{D_1}S^{D_1} + S^*_{D_1}S^*_{D_2}\omega^{D_1D_2} , \tag{4.3}$$

all possible terms in (4.2) can be generated using canonical transformations. The two terms in the action (4.3) is canonically inequivalent as can easily be seen by writing down all possible canonical generators with positive ghost number (there are only four of them). The requirement that S_0 satisfies the master equation implies,

$$\omega^{C_1}{}_D\omega^{DD_2} = 0 . \tag{4.4}$$

This means that $\omega^{C_1}{}_{D_1}$ can be constructed from the null vectors of $\omega^{D_1D_2}$ or vice versa. Furthermore, we observe that the invertibility of one of these two coefficients implies the vanishing of the other. Thus, there exist no solution to the master equation for the action (4.2) where both $\omega^{C_1}{}_{D_1}$ and $\omega^{D_1D_2}$ are invertible, since eq. (4.4) will be one of the equations imposed by $(S,S)_6 = 0$. The superfield content of S can be obtained from S_0 by using the following canonical generator

$$\Gamma = \alpha\,\Gamma_1 + \beta\,\Gamma_2 + \gamma\,\Gamma_3 , \tag{4.5}$$

where

$$\begin{aligned} \Gamma_1 &= S^*_{D_1}\gamma^{D_1}{}_{C_1C_2}R^{C_1}R^{C_2} , \\ \Gamma_2 &= \gamma_{C_1C_2C_3C_4C_5}R^{C_1}R^{C_2}R^{C_3}R^{C_4}R^{C_5} , \\ \Gamma_3 &= \gamma_{C_1D_1D_2}R^{C_1}S^{D_1}S^{D_2} . \end{aligned} \tag{4.6}$$

Although Γ_1, Γ_2 and Γ_3 are not the only possible choice in $n = 6$ using only positive ghost number superfields — they are sufficient. Note that all the coefficients $\gamma^{D_1}{}_{C_1C_2}$, $\gamma_{C_1C_2C_3C_4C_5}$, $\gamma_{C_1D_1D_2}$ and the even and real parameters α, β, γ are functions of the ghost number zero superfield Q^B. An identification of the solution to the master equation $(S,S)_6 = 0$ can be done by comparing the canonically transformed action S_Γ with S. We find,

$$
\begin{aligned}
\omega^{C_1}{}_{C_2C_3} &= \alpha\omega^{C_1}{}_{D}\gamma^{D}{}_{C_2C_3}\,, \\
\omega^{D_1}{}_{C_1C_2C_3} &= -2\alpha^2\gamma^{D_1}{}_{C_1C}\omega^{C}{}_{D}\gamma^{D}{}_{C_2C_3}\,, \\
\omega^{D_1}{}_{C_1D_2} &= -2\alpha\gamma^{D_1}{}_{C_1C}\omega^{C}{}_{D_2} - 4\gamma\omega^{D_1D}\gamma_{DD_2C_1}\,, \\
\omega_{D_1D_2D_3} &= -\gamma\gamma_{D_1D_2C}\omega^{C}{}_{D_3}\,, \\
\omega_{D_1D_2C_1C_2} &= -\gamma\alpha\gamma_{D_1D_2C}\omega^{C}{}_{D}\gamma^{D}{}_{C_1C_2} - 4\gamma^2\gamma_{C_1D_1D}\omega^{DD'}\gamma_{D'C_2D_2} + \\
&\quad + 4\alpha\gamma\gamma_{C_1D_1D}\gamma^{D}{}_{C_2C}\omega^{C}{}_{D_2}\,, \\
\omega_{C_1C_2C_3C_4C_5C_6} &= -5\alpha\beta\gamma_{C_1C_2C_3C_4C}\omega^{C}{}_{D}\gamma^{D}{}_{C_5C_6}\,, \\
\omega_{C_1C_2C_3C_4D_1} &= -5\beta\gamma_{C_1C_2C_3C_4C}\omega^{C}{}_{D_1} + 4\gamma\alpha^2\gamma_{C_1D_1D}\gamma^{D}{}_{C_2C}\omega^{C}{}_{D'}\gamma^{D'}{}_{C_3C_4}\,.
\end{aligned}
\tag{4.7}
$$

The solution given here is for the sake of clarity given for the case when all the superfields $\{S^D, R^C, Q^B\}$ have even parity. For the complete solution with arbitrary parities on the superfields we refer to appendix D. Given a classical model fulfilling the restrictions given above, a quantum master action having (4.1) as its limit is defined by (4.2) and the solution to the coefficients (4.7). One can extend this model by introducing higher order ghost number superfields into the action, and for original theories possessing a high degree of reducibility this is indeed necessary in order to construct the quantum theory. The reason we omitted fields of form degree 6 was that it will introduce superfields with negative ghost number into the theory, due to the relation

$$gh_{\#}K^P + gh_{\#}K^*_P = n - 1\,. \tag{4.8}$$

In that case the master action and the canonical generators are allowed to contain an arbitrary number of superfields and for brevity we choose here to solve for a model excluding these superfields. Extensions to ghost number 6 superfields and higher does not bring in any extra complications with respect to how the solutions are constructed. All these models will have the same classical limit since all the terms containing negative ghost number superfields will be set to zero according to (2.15). The reason for also excluding the superfields Q^B, with ghost number five, is that they couple to all the coefficients ω in the master equation, since they are functions of the scalar superfields Q^B. This implies in general that one must solve complicated equations for the coefficients in S_0. Finding a way to construct solutions in that case would be a natural generalization of the method described above.

Note that the solution (4.7) is derived under the assumption of non-vanishing coefficients in (4.2). For example, a local action with only the $\omega^{C_1}{}_{D_1D_2}$ term non zero is obviously not canonically equivalent to the local action (4.3). The question of which actions of the form (4.2) that are canonically equivalent to simpler actions like (4.3), is in general very difficult to answer. One rigorous but difficult way to investigate such a classification would be to study the cohomology of the BRST-operator $D = (S, .)$ along the lines of Henneaux et al. in for example [24, 30, 31]

At the classical level, for every given model, the possible number of interaction terms involving fields of higher form degree[4] are very few — and this for obvious reasons. This fact explains in part the reason why some of the most interesting models studied in the literature, such as Yang-Mills, BF-theories and Chern-Simons, exhibit absence of fields of higher form degree.

5. Gauge structure of the master action

In this section we investigate the connection between the master action and the corresponding classical action. In [17] it was shown that the Σ variations of the superfields are given by,

$$\begin{aligned} \delta_\Sigma K^P &= \left(\Sigma, K^P\right) = (-1)^n \left(DK^P - (S, K^P)_n\right), \\ \delta_\Sigma K^*_P &= \left(\Sigma, K^*_P\right) = (-1)^n \left(DK^*_P - (S, K^*_P)_n\right). \end{aligned} \tag{5.1}$$

These variations can be used to determine the gauge transformations of the classical model $S_{\rm Cl}$ corresponding to the master action Σ. This is done by applying the rules (2.15) to the Σ-variations (5.1) and then replacing *each* k-form field (K^P reduces to a k-form field) in every term by a $(k-1)$-form gauge parameter, one at a time. In the case of 0-form fields (scalars) the corresponding gauge parameter is zero. For example, from the following Σ-variation of a field S^*_D in $n = 4$,

$$\delta_\Sigma S^*_D = (\Sigma, S^*_D) = DS^*_D - T^*_{E_1}\omega^{E_1}{}_{E_2E_3D_1D}T^{E_2}T^{E_3}S^{D_1}\,, \tag{5.2}$$

where S^*, T^*, T, S are ghost number $3, 2, 1$ and 0-fields, respectively, we derive the gauge transformation,

$$\delta s^*_D = d\tilde{s}^*_D - \tilde{t}^*_{E_1}\omega^{E_1}{}_{E_2E_3D_1D}t^{E_2}t^{E_3}s^{D_1} - 2t^*_{E_1}\omega^{E_1}{}_{E_2E_3D_1D}\tilde{t}^{E_2}t^{E_3}s^{D_1}\,, \tag{5.3}$$

of the corresponding classical 3-form field, s^*_D. Above, the tilde quantities denotes the $(k-1)$-form gauge parameters corresponding to the k-form fields without tilde. Note, that there is no gauge parameter corresponding to the scalar field s^D in (5.3). In section 5.1 below, explicit examples of gauge transformations derived from (5.1) is given.

Now, one might ask the question: given a classical action with certain interaction terms — what class of master actions reduce to this classical action in the limit (2.15)? It turns out that this class is defined by a specific choice of the parities of the superfields entering the master action. These constraints originates from the symmetries of terms in the master action which are of order quadratic or higher in any specific superfield — which implies, since we always can commute these fields in the graded sense, that the coefficients must

[4]Higher meaning of degree $n-1$ or n where n is the dimension of the manifold on which the original model is formulated.

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possess a (graded) symmetry in order to be non vanishing. It is obvious that it suffices to look at quadratic terms in order to derive the parity constraints. Let us look at the terms containing associated superfields K^* first — for example a term

$$K^*_{P_1} K^*_{P_2} \omega^{P_1 P_2}\,, \tag{5.4}$$

implies the following symmetry of the coefficient,

$$\omega^{P_1 P_2} = (-1)^{(\epsilon_{P_1}+1+n)(\epsilon_{P_2}+1+n)} \omega^{P_2 P_1}\,. \tag{5.5}$$

This term reduces to

$$k^*_{P_1} \wedge k^*_{P_2} \omega^{P_1 P_2}\,, \tag{5.6}$$

where according to form degree — and parity description of (2.15)

$$k^*_{P_1} \wedge k^*_{P_2} = (-1)^{(\epsilon_{P_1}+k)(\epsilon_{P_2}+k)+n+1+k} k^*_{P_2} \wedge k^*_{P_1}\,. \tag{5.7}$$

Above, k denotes the form degree of the corresponding field k^P. In deriving (5.7) we used the following property of two $\mathbb{Z}_2$-graded r- and s-forms A and B with parities a and b, respectively,

$$A \wedge B = B \wedge A(-1)^{ab+rs}\,. \tag{5.8}$$

Thus if the coefficient $\omega^{P_1 P_2}$ is to be non vanishing in the classical action for both the symmetries defined by (5.5) and (5.7), we must have

$$(\epsilon_{P_1} + \epsilon_{P_2})(n+1) = k(\epsilon_{P_1} + \epsilon_{P_2})\,. \tag{5.9}$$

A completely analogous analysis of the field term

$$\omega_{P_1 P_2} K^{P_1} K^{P_2}\,, \tag{5.10}$$

gives the single condition

$$k(\epsilon_{P_1} + \epsilon_{P_2}) = 0\,. \tag{5.11}$$

We conclude that the only way to satisfy all cases (5.9) and (5.11) in all dimensions n, is to require that all fields K^P of a specific $gh_{\#}$ have the same parity,

$$\forall P_1, P_2 : \epsilon_{P_1} + \epsilon_{P_2} = 0\,. \tag{5.12}$$

This requirement ensure the existence of a classical term having the same symmetry as its corresponding term in the master action. It should be noted however, that some terms may still vanish at the classical level due to the algebra structure of the model in question. This is the case for certain terms with Lie-algebra valued fields that is traced over, such as for example n-th power terms of 1-form fields in even dimensions n.

5.1 Classical theory

We will now take a look at how one can derive the gauge transformations for a classical model, from its corresponding master action. Consider the six-dimensional model discussed in section 4 and whose interaction terms are given in (4.3). The master action of this theory is given by

$$\Sigma = \int_{\mathcal{M}} d^n u d^n \tau \Big\{ S^*_D D S^D (-1)^{\epsilon_D} + R^*_C D R^C (-1)^{\epsilon_C} + Q^*_B D Q^B (-1)^{\epsilon_B} - \\ - \left(R^*_C \omega^C{}_D S^D + S^*_{D_1} S^*_{D_2} \omega^{D_1 D_2} \right) \Big\} . \tag{5.13}$$

Note that the coefficients in general depend on the scalar fields Q^B,

$$\omega^C{}_D = \omega(Q)^C{}_D\,, \tag{5.14}$$
$$\omega^{D_1 D_2} = \omega(Q)^{D_1 D_2}\,. \tag{5.15}$$

By performing the reduction (2.15) we obtain the following classical action:

$$\Sigma_{\mathrm{Cl}} = \int \big\{ s^*_D \wedge ds^D (-1)^{\epsilon_D} + r^*_C \wedge dr^C (-1)^{\epsilon_C} + q^*_B \wedge dq^B (-1)^{\epsilon_B} - \\ - \left(r^*_C \wedge \omega^C{}_D \wedge s^D + s^*_{D_1} \wedge s^*_{D_2} \omega^{D_1 D_2} \right) \big\} \,. \tag{5.16}$$

Above, q, r, s, s^*, r^*, q^* are $0,1,2,3,4$ and 5-form fields, respectively. The gauge invariance of the classical model is easily derived from (5.1) and is given by,

$$\delta s^*_D = d\tilde{s}^*_D - \tilde{r}^*_C \omega^C{}_D\,, \tag{5.17}$$
$$\delta s^D = d\tilde{s}^D + 2(-1)^{\epsilon_D+1} \tilde{s}^*_{D_1} \omega^{D D_1}\,, \tag{5.18}$$
$$\delta r^*_C = d\tilde{r}^*_C\,, \tag{5.19}$$
$$\delta r^C = d\tilde{r}^C + (-1)^{\epsilon_C+1} \omega^C{}_{D_1} \tilde{s}^{D_1}\,, \tag{5.20}$$
$$\delta q^*_B = d\tilde{q}^*_B + \tilde{r}^*_C \omega^C{}_D \overleftarrow{\partial}_B\, s^D (-1)^{\epsilon_D \epsilon_B} + \\ + r^*_C \omega^C{}_D \overleftarrow{\partial}_B\, \tilde{s}^D (-1)^{\epsilon_D \epsilon_B} + 2\tilde{s}^*_{D_1} s^*_{D_2} \omega^{D_1 D_2} \overleftarrow{\partial}_B\,, \tag{5.21}$$
$$\delta q^B = 0\,. \tag{5.22}$$

The tilded quantities $\tilde{r}, \tilde{s}, \tilde{s}^*, \tilde{q}^*$ and $\tilde{r}^*$ denotes $0,1,2,3$ and 4-form gauge parameters, respectively. The gauge structure above can be viewed as a consistent deformation of the gauge structure of the abelian BF theory [10],

$$\Sigma_{\mathrm{Cl}} = \int s^*_D \wedge ds^D (-1)^{\epsilon_D} + r^*_C \wedge dr^C (-1)^{\epsilon_C} + q^*_B \wedge dq^B (-1)^{\epsilon_B}\,,$$

possessing the gauge symmetry

$$\delta s^*_D = d\tilde{s}^*_D\,, \qquad \delta s^D = d\tilde{s}^D\,, \tag{5.23}$$
$$\delta r^*_C = d\tilde{r}^*_C\,, \qquad \delta r^C = d\tilde{r}^C\,, \tag{5.24}$$
$$\delta q^*_B = d\tilde{q}^*_B\,, \qquad \delta q^B = d\tilde{q}^B\,. \tag{5.25}$$

In fact, from the relatively recent accomplishments of Ikeda [10, 26] it is clear from the rules (2.15) above that *all* classical models derived from a master action of the form (2.1) represent consistent deformations of abelian BF-theories.

6. Structure of the master action in various dimensions

There are certain general rules and similarities existing in various dimensions for the theories we are studying. The first simple analysis one can do, is to look for what constraints the requirement $gh_{\#}S = n$ impose on the coefficients in the local master action S. A Yang-Mill's term in the action always has one of the following two structures,

$$S_{YM} = K^*_{P_1}\omega^{P_1}{}_{P_2P_3}K^{P_2}K^{P_3}\,, \tag{6.1}$$

or

$$S_{YM} = K^*_{P_1}K^*_{P_2}\omega^{P_1P_2}{}_{P_3}K^{P_3}\,, \tag{6.2}$$

depending in what dimension the theory is formulated. The master equation will impose Jacobi-identities for the coefficients $\omega^{P_1}{}_{P_2P_3}(\omega^{P_1P_2}{}_{P_3})$, thus demanding them to belong to a Lie superalgebra. We observe from the superfield ghost number relation

$$gh_{\#}K^P + gh_{\#}K^*_P = n-1\,, \tag{6.3}$$

that one of the superfields in S_{YM} always have ghost number one, implying that we always can identify the corresponding field with a 1-form gauge field. It follows that terms with the structure S_{YM} are allowed in all dimensions. The existence of a group metric requires terms of one of the following two forms in the master action,

$$S_{GM} = \omega_{P_1P_2}K^{P_1}K^{P_2}\,, \tag{6.4}$$

or

$$S_{GM} = K^*_{P_1}K^*_{P_2}\omega^{P_1P_2}\,. \tag{6.5}$$

This fact follows from the requirement that it must be a quadratic term with $gh_{\#} = n$ which couples, via the master equation, to any of the Lie superalgebra terms (6.1) and (6.2). A term with one K- and one K^*-superfield is ruled out due to the $gh_{\#}(S) = n$ condition.

The ghost number requirement for the terms (6.4) and (6.5) together with a specific term S_{YM}, yields $n = 2$ or $n = 4$. Thus, a quadratic coefficient in the action can only have a group metric interpretation in the first two even dimensions. In these two cases, an invertible $\omega_{P_1P_2}$ or $\omega^{P_1P_2}$ can be used to raise and lower the indices of the Lie superalgebra coefficients (6.1) and (6.2).

The number of possible (positive ghost number) interaction terms that can exist in a master action Σ in dimension n, is given by the function $T(n)$,

$$T(n) = \begin{cases} P(n)\,, & n \text{ even}\,, \\ P(n) + P([\frac{n}{2}]+1) + \delta_{n3} - \delta_{n1}\,, & n \text{ odd}\,. \end{cases} \tag{6.6}$$

Above $P(n)$ denotes the $Partition function$ for the integer n and $[n/2]$, the integer part of $n/2$. For the first 11 integers $T(n)$ and $P(n)$ is given by,

n	1	2	3	4	5	6	7	8	9	10	11
$T(n)$	1	2	6	5	10	11	20	22	37	42	67
$P(n)$	1	2	3	5	7	11	15	22	30	42	56

The relatively large number of possible terms in the master actions in odd dimensions, stems from the fact that it always exist superfields K and K^* with the same ghost number when n is odd, thus leading to more possible combinations of interaction terms. If negative ghost number superfields are included, the number of possible terms is of course infinite.

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A. Notation

$$
\begin{aligned}
n &= \text{dimension of the base manifold of the original theory}\,,\\
K &= \text{collective label of superfields}\,,\\
K^* &= \text{collective label of associated superfields to K}\,,\\
K^P &= \text{arbitrary superfield}\,,\\
K^*_P &= \text{arbitrary associated superfield}\,,\\
\epsilon\left(K^P\right) &:= \epsilon_P = \text{Grassmann parity of superfield}\, K^P\,,\\
\epsilon(K^*_P) &= \epsilon_P + n + 1 = \text{Grassmann parity of associated superfield}\, K^*_P\,,\\
gh_{\#} &= \text{ghost number}\,,\\
\epsilon(F) &= \text{Grassmann parity of object F}\,.
\end{aligned}
$$

B. Conventions

Ghost numbers of the superfields are chosen according to the convention

$$gh_{\#}K^*_P \geq gh_{\#}K^P\,.$$

The ghost number determines also the labeling and the indices of the superfields according to,

superfield K^*	M	N	O	P	Q	R	S	T	U	V	X	Y	Z
$gh_{\#}$	9	8	7	6	5	4	3	2	1	0	-1	-2	-3
index	Z	X	Y	A	B	C	D	E	F	G	H	I	J

The major advantage of this labeling of the superfields is that one can directly read off what kind of superfields that should be multiplied with any of the coefficients, just by looking at their label and index structure. E.g. a $\omega^{C_1}{}_{D_1}$-term will always multiply the superfields as $R^*_{C_1}\omega^{C_1}{}_{D_1}S^{D_1}$ in any dimension. Furthermore, one is not confused when interpreting expressions in different dimensions, since an associated superfield K^* of a specific ghost number will have the same label in all dimensions — for example, the associated superfield S^*_D will always denote a ghost number 3 field, regardless of the dimension of the supermanifold it lives on. All objects are ordered with the associated superfields to the left and the superfields to the right.

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C. Canonical generator for superfields

Theorem. *Given a canonical generator* Γ *of the form*

$$\Gamma = K^*_{P_1} K^*_{P_2} \dots K^*_{P_n} \Gamma^{P_1 P_2 \dots P_n}{}_{P_{1'} P_{2'} \dots P_{n'}} K^{P_{1'}} K^{P_{2'}} \dots K^{P_{n'}} \,, \tag{C.1}$$

where Γ *does not contain both a specific superfield and its associated superfield simultaneously. This will generate a canonical transformation of the superfields and the associated superfields which is first order in the real and even parameter* γ *that parametrizes the transformation.*

Proof. Consider arbitrary superfields K^P and associated superfields K^*_P. Let $\bar{K}^P$ and $\bar{K}^*_P$ denote the corresponding canonically transformed superfields. Since Γ does not contain both K^P and K^*_P simultaneously, we have $(\Gamma,\Gamma)_n = 0$. This implies that Γ contracted with K^P and K^*_P must satisfy $((K^P,\Gamma)_n,\Gamma)_n = 0$ and $((K^*_P,\Gamma)_n,\Gamma)_n = 0$. Thus, either we have the superfields transforming as $\bar{K}^P = K^P$ and $\bar{K}^*_P = e^{\gamma \mathrm{ad}\,\Gamma} K^*_P = K^*_P + \gamma (K^*_P,\Gamma)_n$ (since higher order terms vanish) or $\bar{K}^P = e^{\gamma \mathrm{ad}\,\Gamma} K^P = K^P + \gamma(K^P,\Gamma)_n$ and $\bar{K}^*_P = K^*_P$, or they stay untransformed depending on whether Γ contains the superfields or not. Notice that the n-bracket is preserved by the canonical transformations generated by Γ, $(\bar{K}^P, \bar{K}^*_{P'})_n = \delta^P{}_{P'}$.

It follows as a trivial corollary that for every local action S_0 with a finite number of polynomial terms, $S_\Gamma = e^{\mathrm{ad}\,\Gamma} S_0$ will also consist of a finite number of terms.

D. Solution to the six-dimensional model

Below is given the full solution (with arbitrary parities of the fields) to the master equation for the $n = 6$ models studied in section 4. Here we denote Grassmann parities as $\epsilon_C = C$,

$$\begin{aligned}
\omega^{C_1}{}_{C_2C_3} &= \alpha \omega^{C_1}{}_D \gamma^D{}_{C_2C_3} \,, \\
\omega^{D_1}{}_{C_1C_2C_3} &= -2\alpha^2 \gamma^{D_1}{}_{C_1C} \omega^C{}_D \gamma^D{}_{C_2C_3} (-1)^{C_1(C+C_2+C_3+1)} \,, \\
\omega^{D_1}{}_{C_1D_2} &= -2\alpha\gamma^{D_1}{}_{C_1C}\omega^C{}_{D_2}(-1)^{C_1(C+D_2+1)} - \\
&\quad - 4\gamma\omega^{D_1D}\gamma_{DD_2C_1}(-1)^{D_1(D+1)+C_1(D_2+D)+D_2D} \,, \\
\omega_{D_1D_2D_3} &= -\gamma\gamma_{D_1D_2C}\omega^C{}_{D_3}(-1)^{(D_1+D_2)(C+D_3+1)} \,, \\
\omega_{D_1D_2C_1C_2} &= -\gamma\alpha\gamma_{D_1D_2C}\omega^C{}_D\gamma^D{}_{C_1C_2}(-1)^{(D_1+D_2)(1+C+C_1+C_2)} - \\
&\quad - 4\gamma^2\gamma_{C_1D_1D}\omega^{DD'}\gamma_{C_2D_2D'} \times \\
&\qquad \times (-1)^{D(D'+1)+(D'+C_2+D_2)(C_1+D_1)+C_1D_1+D_2(C_1+C_2)} + \\
&\quad + 4\alpha\gamma\gamma_{C_1D_1D}\gamma^D{}_{C_2C}\omega^C{}_{D_2}(-1)^{C_1(D_1+D_2)+(C_1+D_1)(D+C_2+D_2+1)+C_2(C+1)} \,, \\
\omega_{C_1C_2C_3C_4C_5C_6} &= -5\alpha\beta\gamma_{C_1C_2C_3C_4C}\omega^C{}_D\gamma^D{}_{C_5C_6}(-1)^{(C_1+C_2+C_3+C_4)(C+1+C_5+C_6)} \,, \\
\omega_{C_1C_2C_3C_4D_1} &= -5\beta\gamma_{C_1C_2C_3C_4C}\omega^C{}_{D_1}(-1)^{(C+D_1+1)(C_1+C_2+C_3+C_4)} + \\
&\quad + 4\gamma\alpha^2\gamma_{C_1D_1D}\gamma^D{}_{C_2C}\omega^C{}_{D'}\gamma^{D'}{}_{C_3C_4} \times \\
&\qquad \times (-1)^{(C_1+D_1)(D+C_2+1+C_3+C_4)+C_2(C+C_3+C_4+1)} \,.
\end{aligned} \tag{D.1}$$

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